

ATUL VIDYALAYA
FIRST PRELIMINARY EXAMINATION-2012-13
MATHEMATICS

STD: XII Science
DATE: 01/10/2012
SESSION : I

MM: 100
TIME: 3 hrs

GENERAL INSTRUCTION
(Three hours)

(Candidate are allowed additional 15 minutes for **only** reading the paper .
They must **NOT** start writing during this time)

There will be one paper of **three** hours duration of 100 marks. The syllabus is divided into three sections A , B and C. Section A is compulsory for all candidates. Candidates will have choice of attempting questions from **either** from Section B or Section C.

Section A(80 marks) will consists of 9 questions. Candidate will be required to answer **Question -1 (Compulsory)** and **five** out of the rest of the eight question.

Section B/C(20 marks) Candidate will be required to answer two questions out of three from either Section B or Section C.

Section – A

Question 1.

i) Solve for x : $\cos(\sin^{-1} x) = \frac{1}{7}$. [3]

ii) Prove that : $\tan^{-1} \frac{2}{3} = \frac{1}{2} \tan^{-1} \frac{12}{5}$.
[3]

iii) Solve the differential equation $x^2(y+1)dx + y^2(x-1)dy = 0$.
[3]

iv) For matrix $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$, find the number K_1 and K_2 such that $A^2 + K_1A + K_2I = 0$.
Hence , find A^{-1} . [3]

v) Find the modulus and argument of the complex number $\frac{5-i}{2-3i}$. [3]

vi) Evaluate : $\lim_{x \rightarrow 0} \frac{3 \sin 2x - 2 \sin 3x}{x^3}$.
[3]

vii) Two dice are rolled once . Find the probability that the numbers on the two dice are different . What is the probability that the total is at least 4 ? [3]

viii) If p and q be the imaginary cube roots of unity , prove that $p^2 + q^2 - pq = -2$. [3]

ix) Show that the straight line $5x + 12y = 9$ touches the hyperbola $x^2 - 9y^2 = 9$ and find the point of contact . [3]

x) Differentiate $\tan^{-1} \left(\frac{3x - x^3}{1 - 3x^2} \right)$ with respect to x .
[3]

Question 2.

i) If x, y, z are all different and $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$, show that $xyz = -1$.
[5]

ii) Using matrices , solve the following homogeneous equations : [5]

$$\begin{aligned}
 3x + 2y + 7z &= 0 \\
 \text{EMBED Equation.DSMT4 } 4x - 3y - 2z &= 0 \\
 5x + 9y + 23z &= 0
 \end{aligned}$$

Question 3.

i) If $y = \sqrt{\frac{1-x}{1+x}}$, prove that $(1-x^2)\frac{dy}{dx} + y = 0$. [5]

Atul Vidyalyaya
STD. XII SCI.

Pg.1

Shaping the future
MATHEMATICS

ii) Show that $\frac{1}{2} \tan^{-1} x = \cos^{-1} \sqrt{\frac{1 + \sqrt{1+x^2}}{2\sqrt{1+x^2}}}$. [5]

Question 4

i) Evaluate the following : $\int x \tan^{-1} x dx$ [5]

ii) x, y and z represent three switches in an " ON " position and x', y', z' represent the three switches in an " OFF " position.

Construct a switching circuit representing the polynomial $(x' + y')(x + z') + y'(y + z)$. Using the laws of Boolean algebra, show that the above polynomial is equivalent to $x'z' + y'$ and construct an equivalent switching circuit. [5]

Question 5.

i) Find the area A bounded by the x – axis, part of the curve $y = 1 - \frac{8}{x^2}$ and the ordinate $x = 4$ and $x = 8$. If the straight line passing through the point (a, k) and parallel to the y – axis divides the area A into two parts in the ratio $4 : 5$, Find the value of a . [5]

ii) Using De Moivre's theorem, find the least value of n for which the expression $\left(\frac{1+i}{1-i}\right)^n$ is purely imaginary. [5]

Question 6.

i) Show that the semi-vertical angle of the cone of maximum volume and of given slant height is $\tan^{-1} \sqrt{2}$. [5]

ii) It is given that for the function $f(x) = x^3 - 6x^2 + ax + b$ on $[1, 3]$, Rolle's theorem holds with $C = 2 + \frac{1}{\sqrt{3}}$. Find the values of a and b if $f(x) = f(3) = 0$. [5]

Question 7.

i) The mathematical aptitude scores of 10 Software Computer programmers with their job performance are given below :

Programmers	A	B	C	D	E	F	G	H	I	J
Mathematics Scores	5	1	4	3	0	2	6	8	9	7
Job performance rating	8	16	8	9	5	4	3	8	17	12

Calculate Spearman's rank correlation coefficient. [5]

ii) In a bivariate distribution, it was found $\sigma_x = 3$, the regression line of Y on X is

$8x - 10y + 66 = 0$ while regression line of X on Y is $40x - 18y - 214 = 0$. Calculate $\bar{x}, \bar{y}, \rho(X, Y), Cov.(X, Y)$ and σ_y . [5]

Question 8.

- i) Two unbiased dice are thrown .Find the probability that :
 (a) Neither a doublet nor a total of 8 will appear ;
 (b) The sum of the numbers obtained on the two dice is neither a multiple of 2
 Nor a multiple of 3 . [5]
- ii) A problem in Mathematics is given to 3 students whose chances of solving it are

Atul Vidyalaya

Pg.2

Shaping the future

STD. XII SCI

MATHEMATICS

$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$. What is the probability that the problem is solved ?

Question 9

- i) If $y = A \cos nx + B \sin nx$, prove that $\frac{d^2 y}{dx^2} + n^2 y = 0$. [5]
- ii) Solve the differential equation $(1 + x^2) \frac{dy}{dx} + y = \tan^{-1} x$ [5]

(Answer two questions from either Section B or Section C)

SECTION B

Question 10.

- i) Find the equations of the planes through the intersection of the planes $x + 3y + 6 = 0$ and $3x - y - 4z = 0$ whose perpendicular distance from the origin is equal to 1 . [5]
- ii) Show that the lines $\frac{x-5}{4} = \frac{y-7}{4} = \frac{z+3}{-5}$ and $\frac{x-8}{7} = \frac{y-4}{1} = \frac{z-5}{3}$ intersect . Find their Point of intersection . [5]

Question 11.

- i) Prove by vector method that the perpendicular bisector of the sides of triangle are Concurrent . [5]
- ii) Find the area of a parallelogram whose diagonals are determined by the vectors $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} - 3\hat{j} + 4\hat{k}$. [5]

Question 12.

- i) A random variable x has the following probability distribution :

x	0	1	2	3	4	5	6	7
P(x)	a	4a	3a	7a	8a	10a	6a	9a

- a) Determine the value of a .
- b) Find $P(x < 3), P(x \leq 4), P(0 < x < 5)$.
- c) Give the smallest value of m for which $P(x \leq m) \geq 0.6$. [5]
- ii) Bag I contains 2 white and 3 red balls and bag II contains 4 white and 5 red balls . A bag is taken at random and a ball is drawn from it . If the ball drawn is red , find the probability that it was drawn from bag I . [5]

SECTION C

Question 13.

- i) A bill of ₹ 1,000 drawn on May 7, 2003 for 6 months was discounted on August 29, 2003 for cash payment of ₹ 988. Find the rate of interest charged by the bank. [5]
- ii) A man purchases a house and takes a mortgage on it for ₹ 8,00,000 to be paid off in 12 years by equal annual payments. If the interest rate is 9% per annum compounded annually, what amount will be required to pay each year? [5]

Question 14.

- i) A company has produced x items and the total cost C and total revenue R are given by the equation $C = 100 + 0.015x^2$ and $R = 3x$. Find how many items should be produced to maximize the profit. What is the profit? [5]

Atul Vidyalaya**Pg.3****Shaping the future****STD. XII SCI.****MATHEMATICS**

- ii) A shopkeeper deals in two items – wall hangings and artificial plants. He has ₹ 15000 to invest and a space to store at most 80 pieces. A wall hanging costs him ₹ 300 and an artificial plant ₹ 150. He can sell a wall hanging at a profit of ₹ 50 and an artificial plant at a profit of ₹ 18. Assuming that he can sell all the items that he buys, formulate a linear programming problem in order to maximize his profit. [5]

Question 15.

- i) Using 1985 as base year, the index numbers for the price of a commodity in 1986 and 1987 are 118 and 125. Calculate the index numbers for (i) 1985 and (ii) 1987 if 1986 is taken as the base year. [5]
- ii) Daily absence from a school during 3 weeks is recorded as follows :

	Monday	Tuesday	Wednesday	Thursday	Friday
Week 1	23	28	21	33	40
Week 2	38	52	43	58	63
Week 3	52	54	61	51	51

Draw the graph of these figures. Calculate 5 – day moving averages and plot them on the same graph.

Atul Vidyalaya

Pg.4

Shaping the future