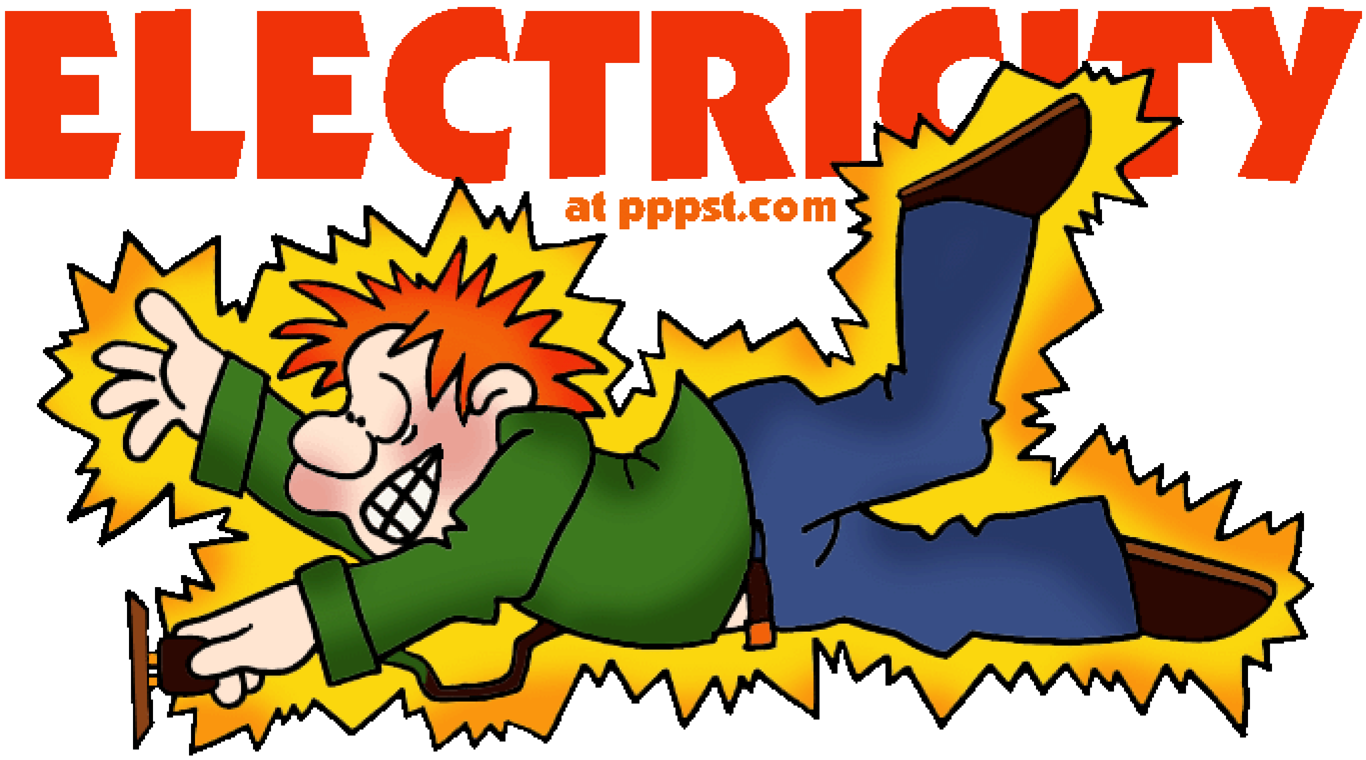

ELECTRICITY
at pppst.com

Electrostatics

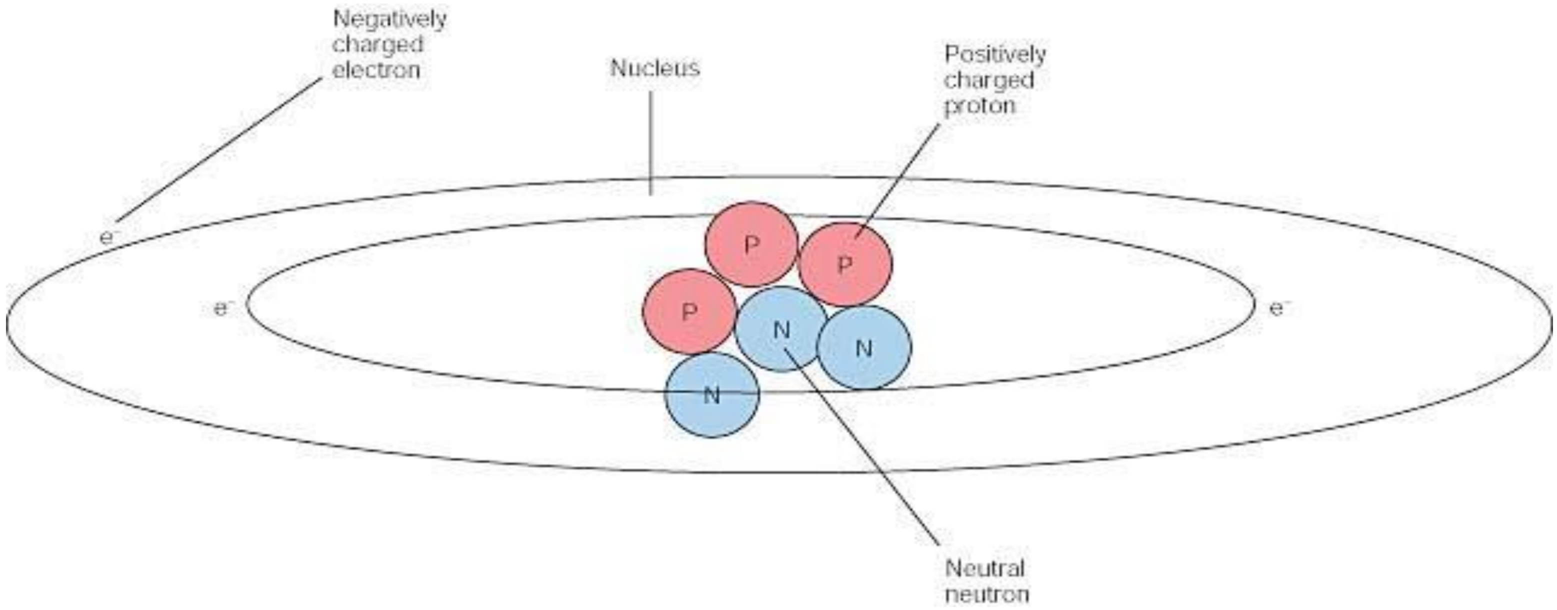
Nay, electrophun!!!

What is electricity?

The collection or flow of electrons in the form of an electric charge

Electric Charge and Electrical Forces:

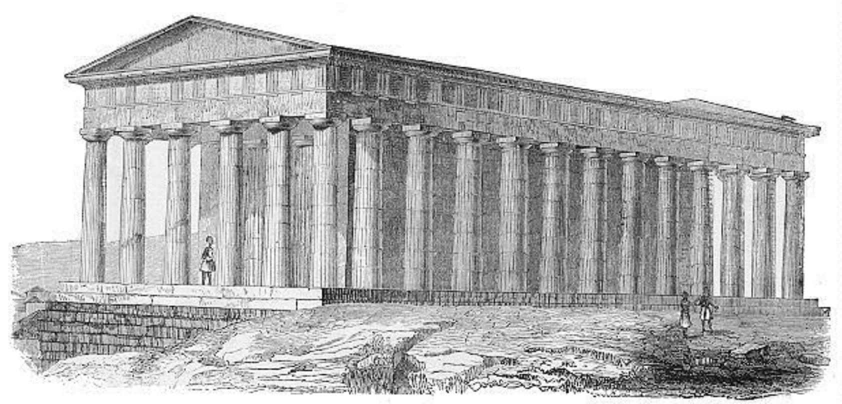
- Electrons have a negative electrical charge.
- Protons have a positive electrical charge.
- These charges interact to create an electrical force.
  - Like charges produce repulsive forces – so they repel each other (e.g. electron and electron or proton and proton repel each other).
  - Unlike charges produce attractive forces – so they attract each other (e.g. electron and proton attract each other).



A very highly simplified model of an atom has most of the mass in a small, dense center called the nucleus. The nucleus has positively charged protons and neutral neutrons. Negatively charged electrons move around the nucleus at much greater distance. Ordinary atoms are neutral because there is a balance between the number of positively charged protons and negatively charged electrons.

History

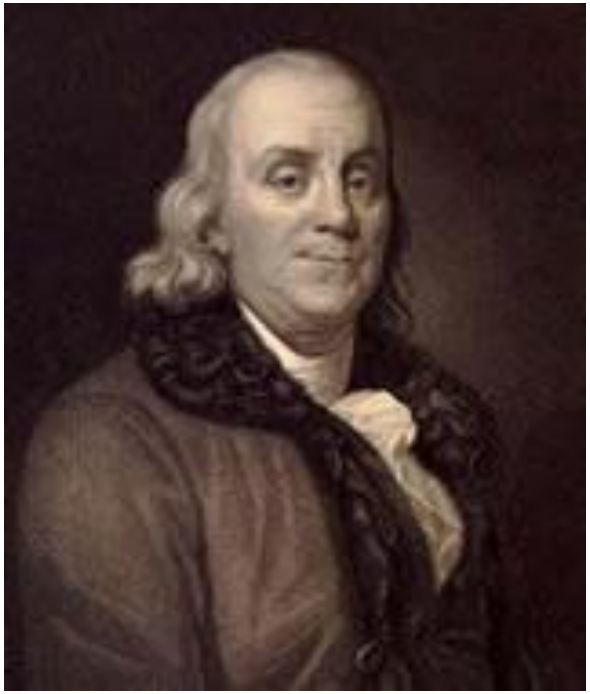
- The word *electricity* comes from the Greek *elektron* which means “amber”.
- The “amber effect” is what we call *static electricity.*

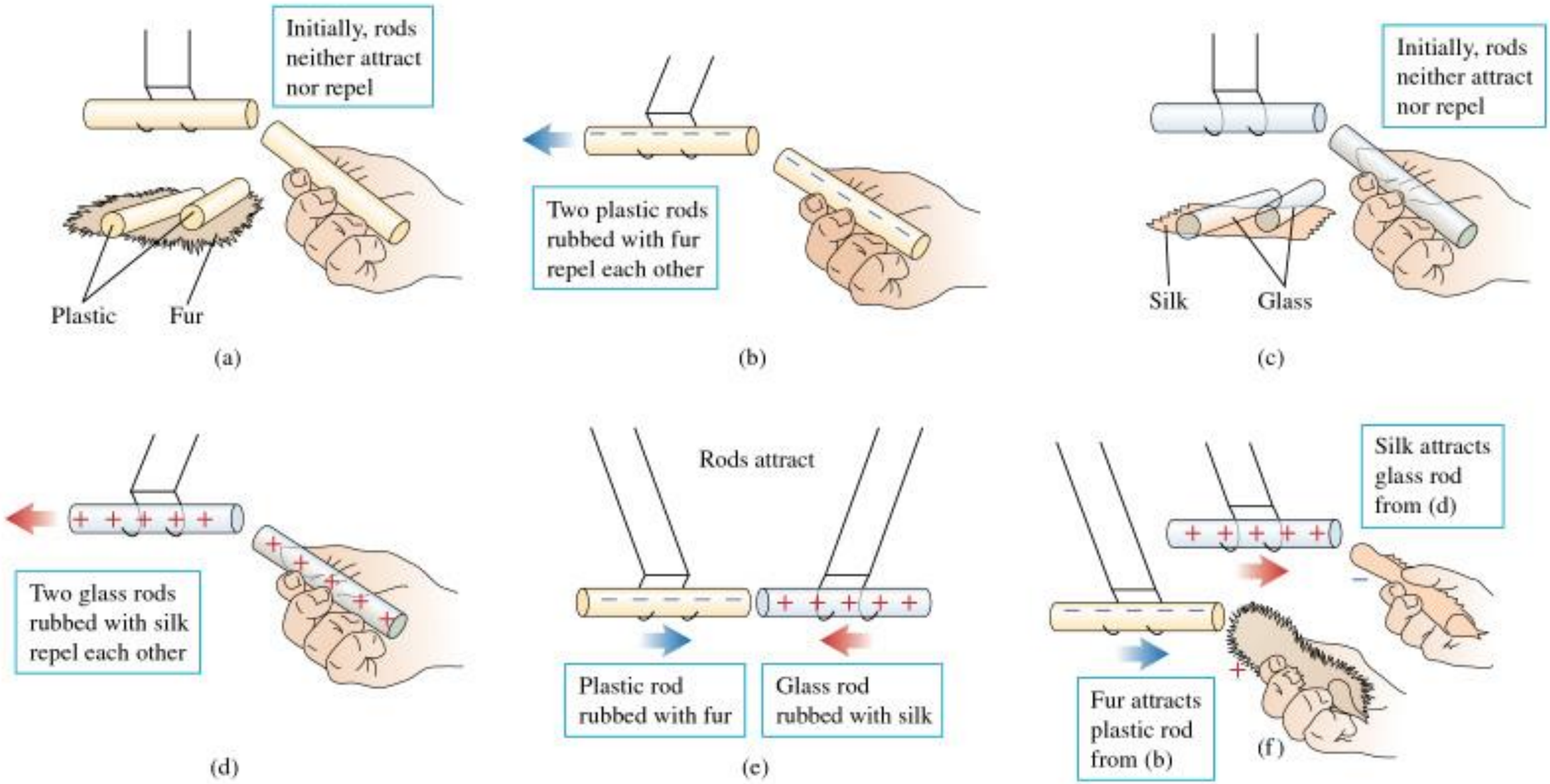


View of the Temple of Theseus at Athens.

History

- Ben Franklin made the arbitrary choice of calling one of the demo situations *positive* and one *negative*.
- He also argued that when a certain amount of charge is produced on one body, an equal amount of the opposite charge is produced on the other body…





Initially, rods neither attract nor repel
Plastic
Fur
(a)
Two plastic rods rubbed with fur repel each other
(b)
Initially, rods neither attract nor repel
Silk
Glass
(c)
Two glass rods rubbed with silk repel each other
(d)
Rods attract
Plastic rod rubbed with fur
Glass rod rubbed with silk
(e)
Silk attracts glass rod from (d)
Fur attracts plastic rod from (b)
(f)

Charge Concepts

- Opposite charges attract, like charges repel.
- Law of Conservation of Charge:
  - The net amount of electric charge produced in any process is zero.
- **Symbol**: q, Q
- **Unit**: C, Coulomb

Elementary Particles

Particle	Charge, (C)	Mass, (kg)
electron	$-1.6x10^{-19}$	$9.109x10^{-31}$
proton	$+1.6x10^{-19}$	$1.673x10^{-27}$
neutron	0	$1.675x10^{-27}$

- If an object has a…
  - \+ charge → it has less electrons than normal
  - – charge → it has more electrons than normal

$$\#electrons = \frac{q_{total}}{1.6x10^{-19}}$$

Ions and Polarity

- If an atom loses or gains valence electrons to become + or – , that atom is now called an *ion*.
- If a molecule, such as H_2O, has a net positive charge on one side and negative charge on the other it is said to be *polar*

Why does…

Chemistry work?

Physics!!!

The electrostatic forces between ions (within molecules) form bonds called ionic bonds…all bonds are ionic; others, like covalent, are to a much lesser degree so that you can ignore the ionic properties of that type of bond.

Why does…

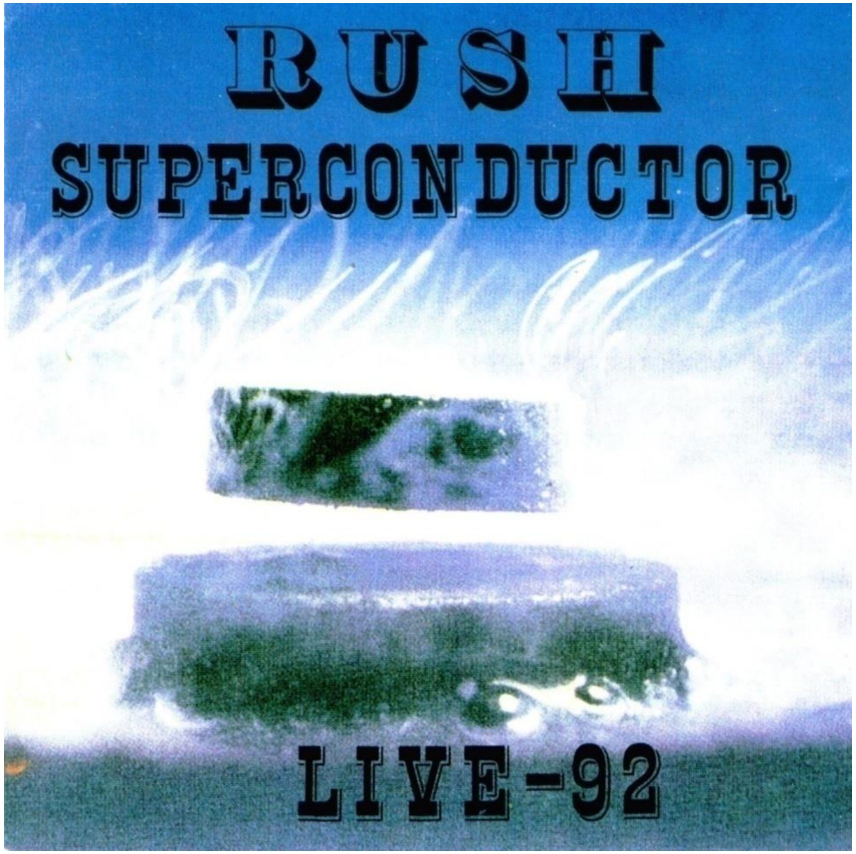
Biology work?

Physics!!!

The intermolecular electrostatic forces between polar molecules make such things as the DNA double helix possible.

Types of materials

1. **Conductor:** a material that transfers charge easily (ex. Metals).
2. **Insulator:** a material that does not transfer charge easily (ex. Nonmetals)
3. *Semiconductors:* somewhere between 1 & 2 (ex. Silicon, carbon, germanium).
4. *Superconductors:* some metals become perfect conductors below certain temperatures



What is a conductor and insulator?

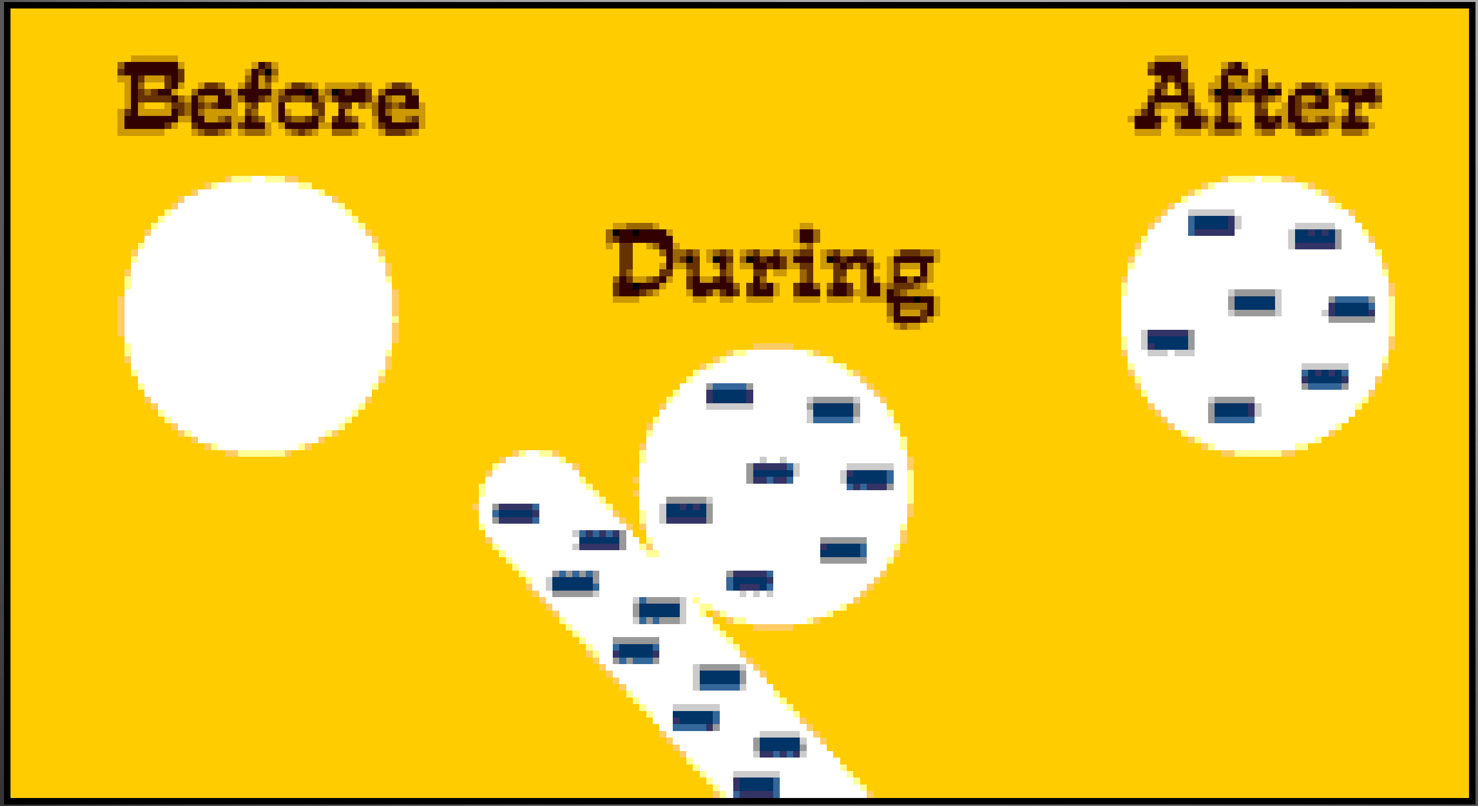
A conductor is a material which allows an electric current to pass. Metals are good conductors of electricity.

An insulator is a material which does not allow an electric current to pass. Nonmetals are good conductors of electricity. Plastic, glass, wood, and rubber are good insulators

Ways to Charge

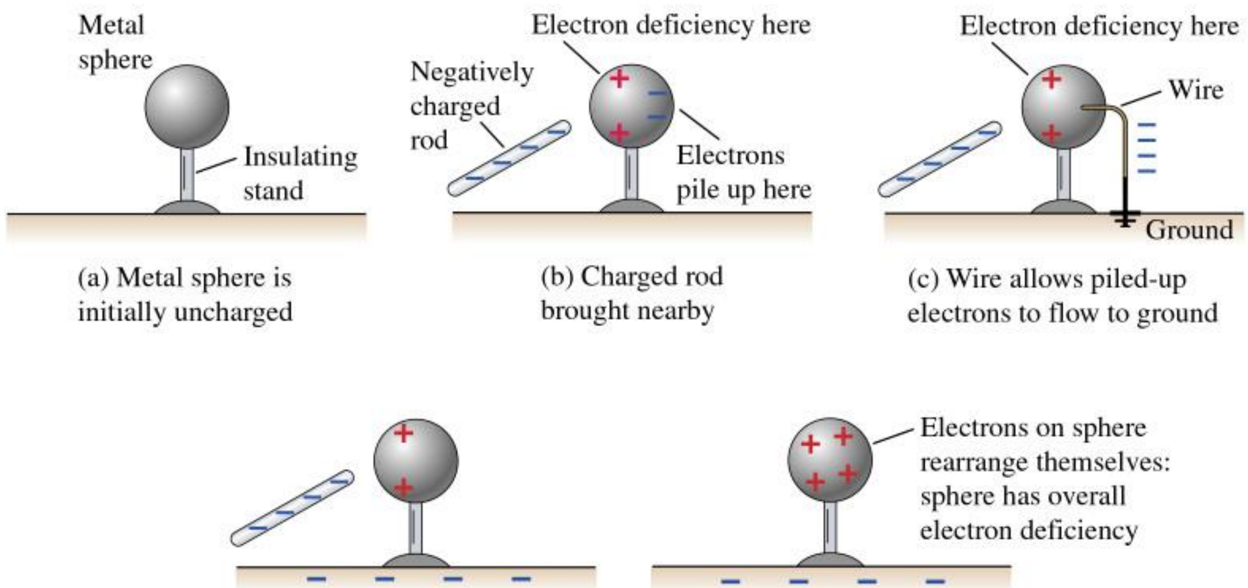
- **By Conduction**: contact occurs between charged object and neutral object.
  - Result: two objects with same charge
- **By induction**: no contact occurs between charged object and neutral object.
  - Result: two objects with opposite charge
- **Credit Card:** You may use Visa, Master Card, or American Express
  - Result: Debt from high interest rates

Conduction



Induction

Polarization



(a) Metal sphere is initially uncharged

(b) Charged rod brought nearby

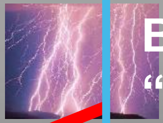
(c) Wire allows piled-up electrons to flow to ground

(d) Wire is disconnected from sphere

(e) Charged rod is removed

Lightning

Becomes very "negative"



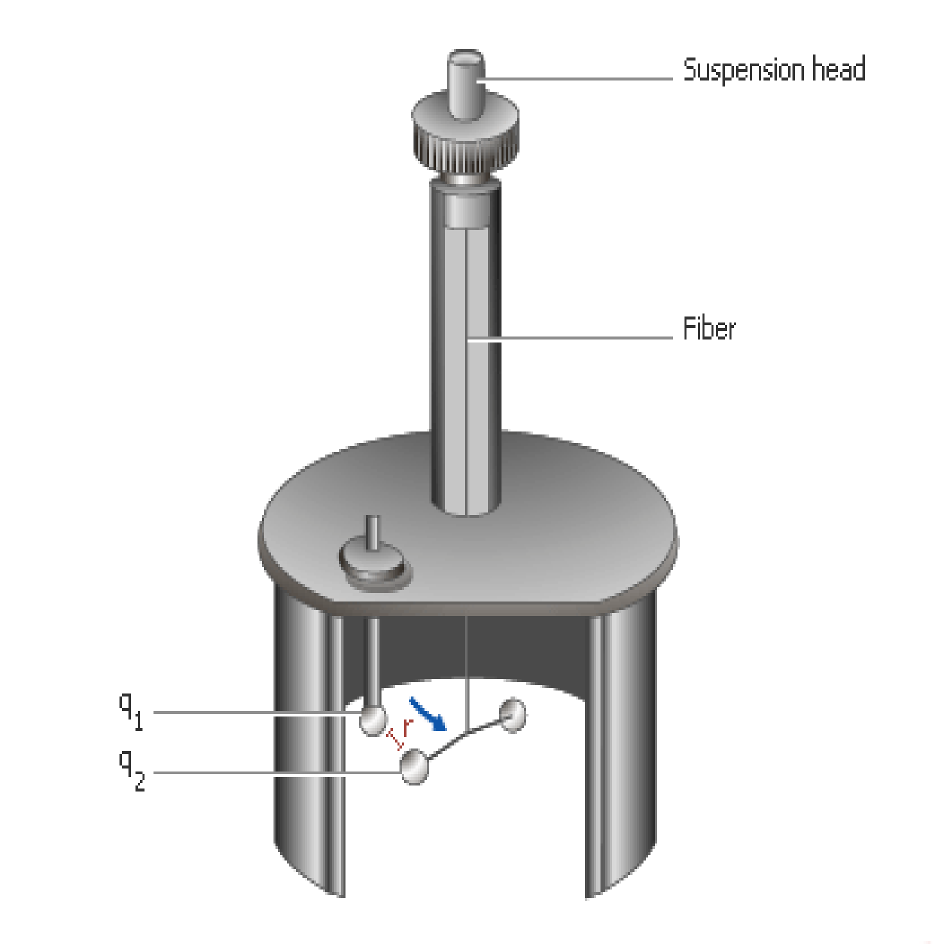
Becomes very "positive"

Electric Force

Coulomb's Law

Using a torsion balance, Coulomb found that: *the electric force between two charges is proportional to the product of the two charges and inversely proportional to the square of the distance between the charges.*

Electric Force



Electric Force

$$\vec{F}_E = k_c \frac{q_1 q_2}{r^2}$$

- q → charge, C
- r → distance between charges, m
- F_E → Electric Force, N → VECTOR
- k_c → coulomb constant, $8.99 \times 10^9 Nm^2/C^2$

Coulomb Constant

$$k_c = 8.99 \times 10^9 \frac{Nm^2}{C^2}$$

$$or$$

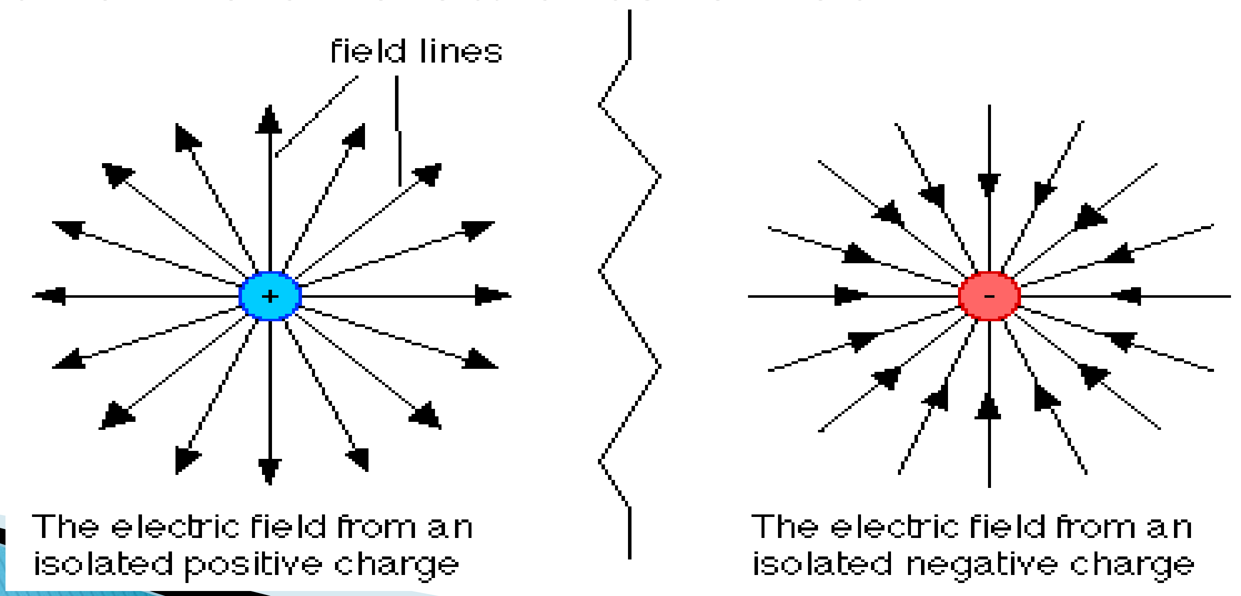
$$k_c = \frac{1}{4\pi\varepsilon_0}$$

$$\varepsilon_0 = 8.854 \times 10^{-12} \frac{C^2}{Nm^2}$$

Electric Field

The electric force is a field force, it applies force without touching (like the gravitational force)

In the region around a charged object, an *Electric Field* is said to exist



Electric Field

Rules for Drawing Electric Field Lines

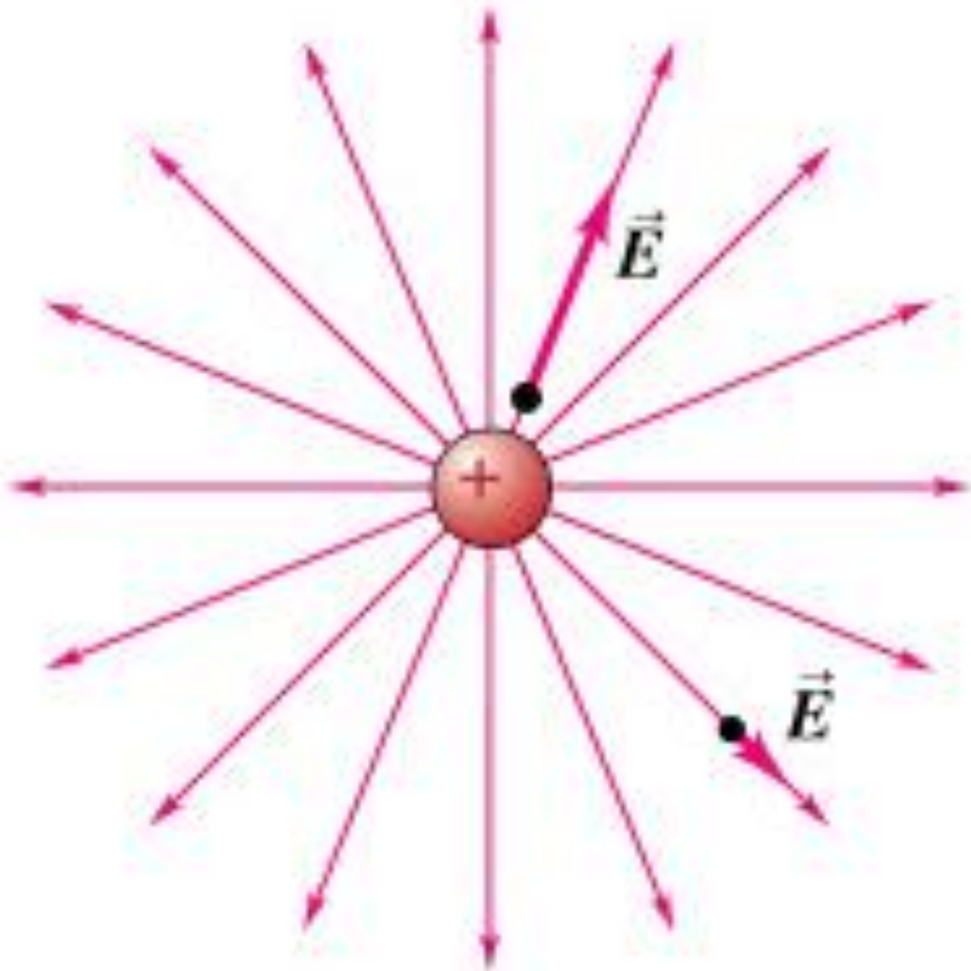
1. The lines must originate on a positive charge (or infinity) and end on a negative charge (or infinity).
2. The number of lines drawn leaving a positive charge or approaching a negative charge is proportional to the magnitude of the charge.
3. No two field lines can cross each other.
4. The line must be perpendicular to the surface of the charge

Rules for Drawing Electric Field Lines (Cont…)

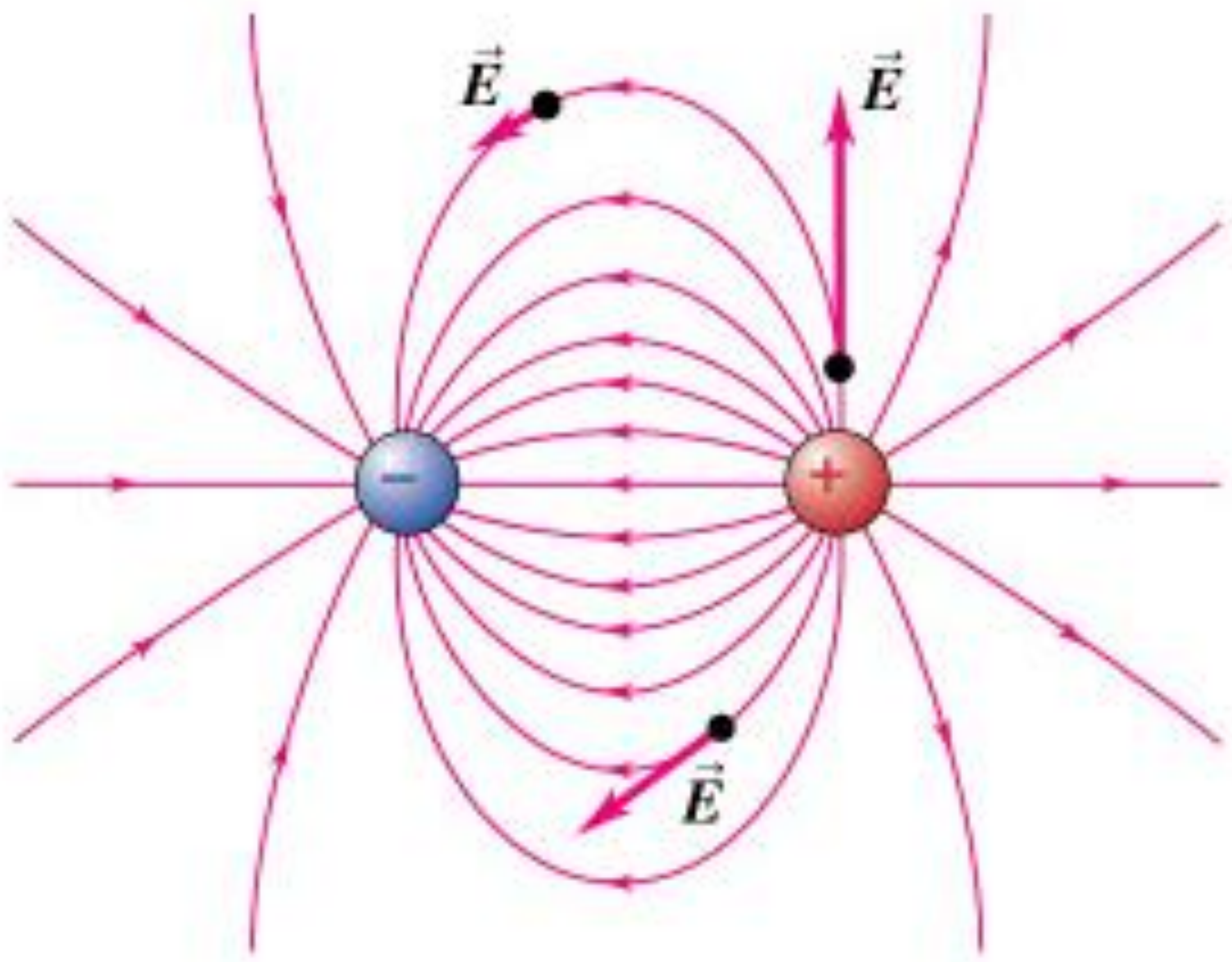
- The tangent to the field lines give the direction of electric intensity.
- Field lines never meet.
- We can draw lines through every point on the field.
- No field lines pass through a conductor.
- The field lines expand sidewise. This helps to understand repulsion.
- The field lines contract lengthwise This helps to understand attraction.

Conductors in Electrostatic Equilibrium

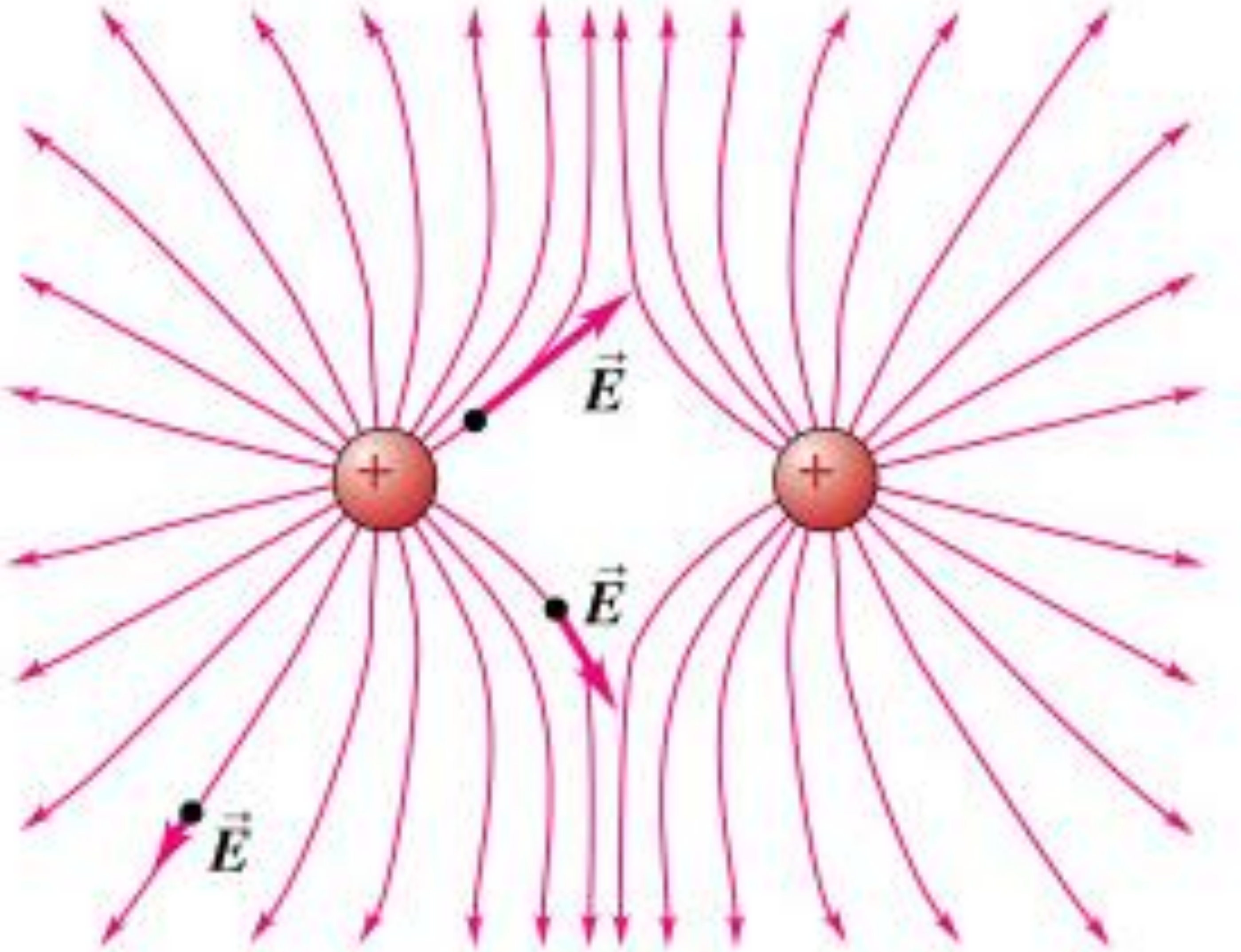
1. The electric field is zero everywhere inside a conductor.
2. Any excess charge on an isolated conductor resides entirely on the outside surface of the conductor.
3. The electric field just outside the charged conductor is perpendicular to the conductor's surface.
4. On an irregularly shaped conductor, charge tends to accumulate where the radius of curvature is the smallest, i.e. AT SHARP POINTS.



(a) A single positive charge
(compare Figure 21.16)

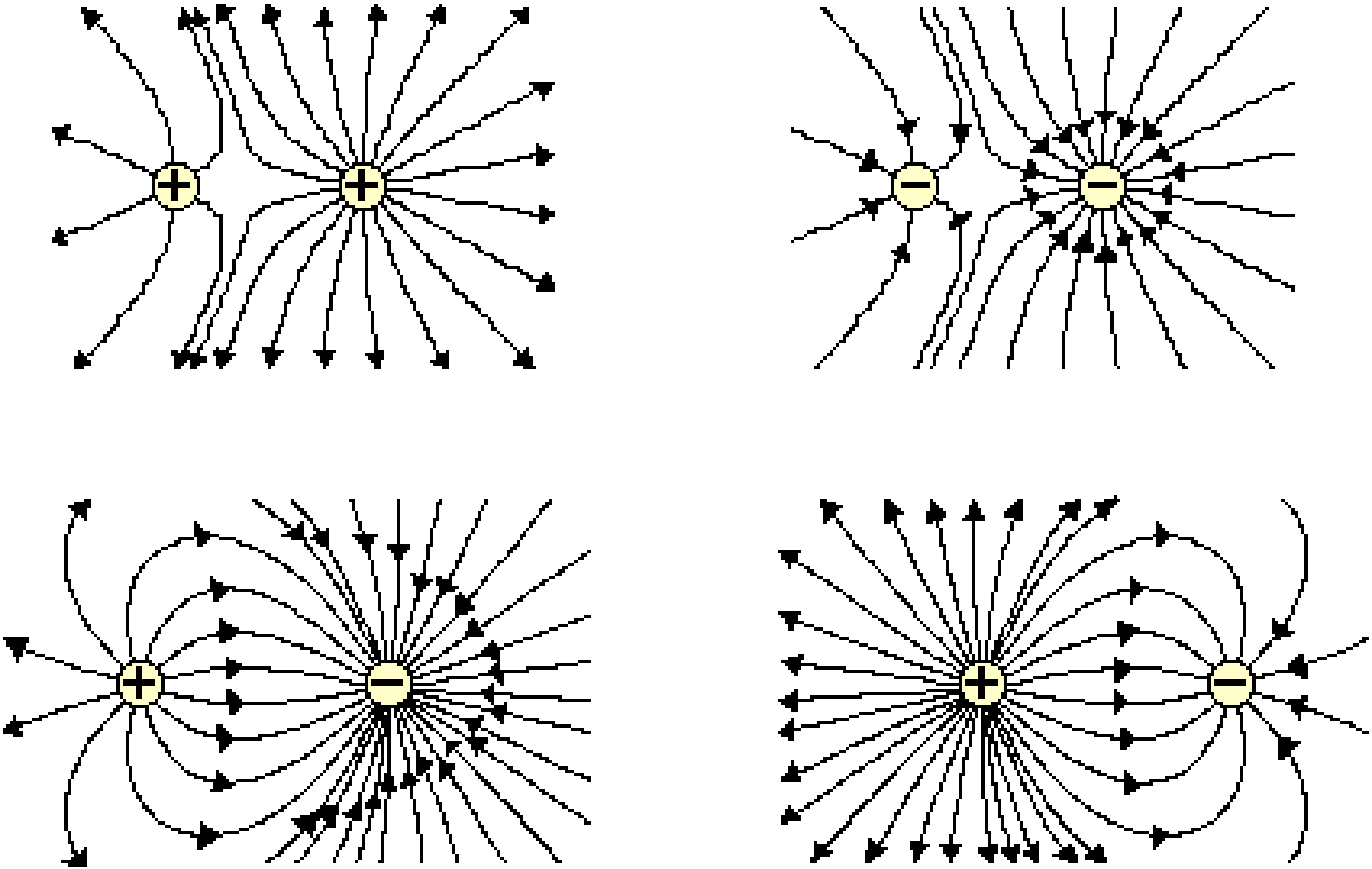


(b) A positive charge and a negative charge of equal magnitude (an electric dipole)

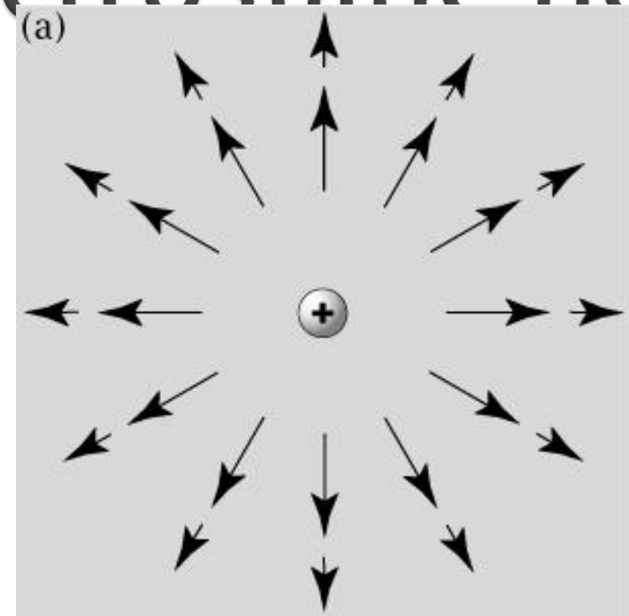


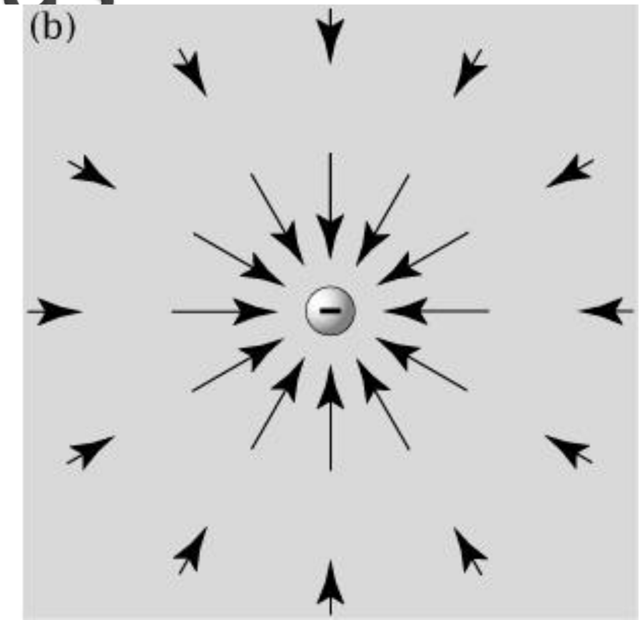
(c) Two equal positive charges

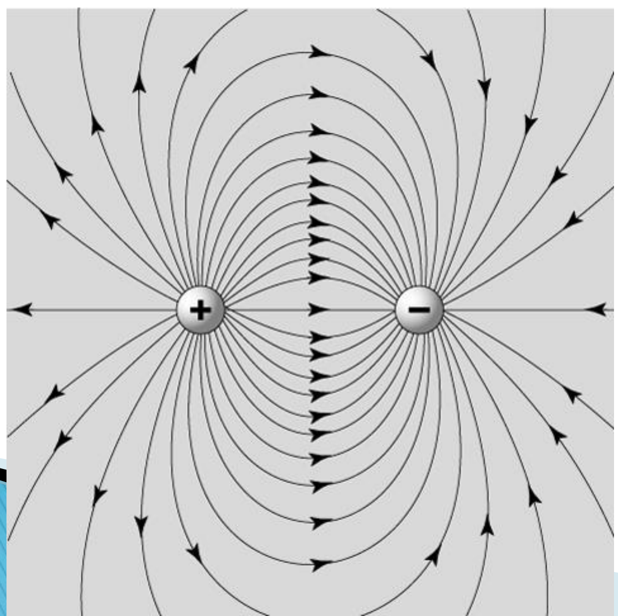
Electric Field Line Patterns for Objects with Unequal Amounts of Charge

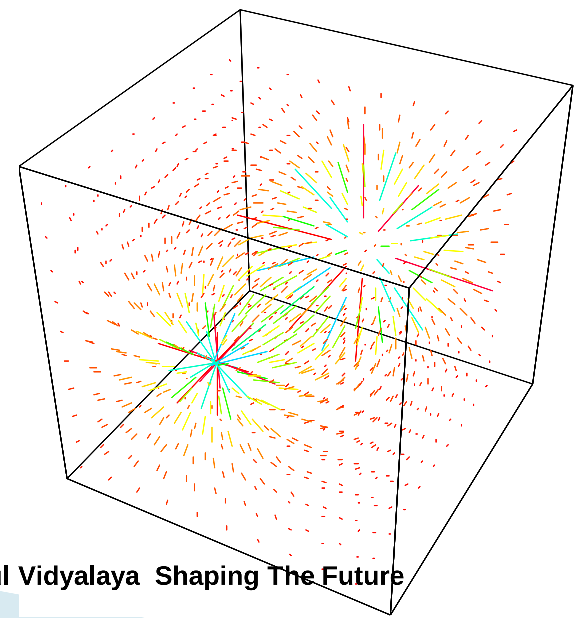


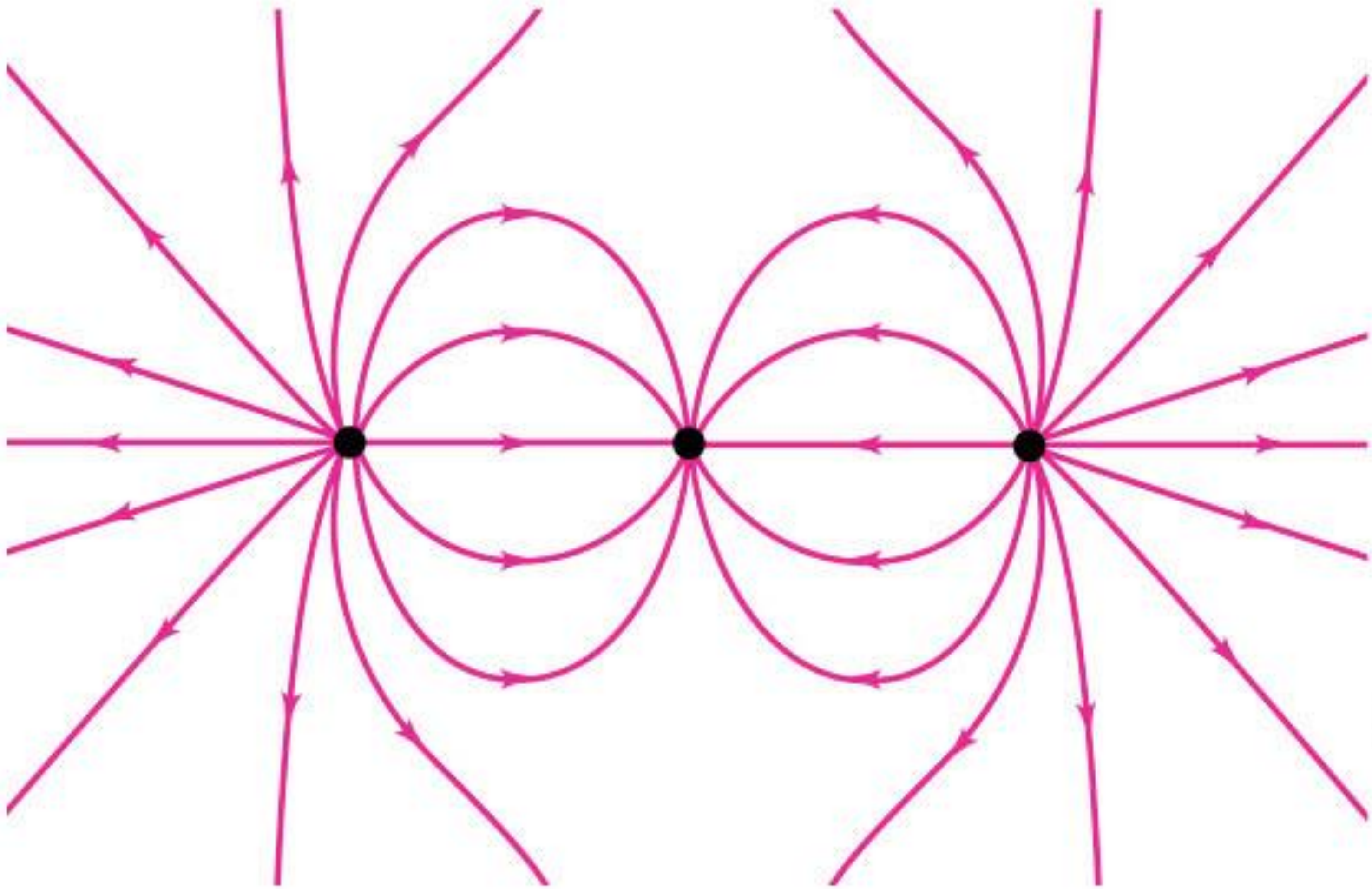
Electrostatic fields











Electric Field

$$\vec{E} = \frac{\vec{F}_E}{q_0} \quad becomes \quad E = k_c \frac{q}{r^2}$$

- E → electric field strength, N/C → VECTOR
- q_0 → + test charge, C
- q → charge producing field, C
- r → distance between charges, m
- F_E → Electric Force, N → VECTOR
- k_c → coulomb constant, $8.99 \times 10^9 Nm^2/C^2$

E–Field vs g–field

$$E - Field$$

$$g - field$$

$$\vec{E} = \frac{\vec{F}_0}{q_0}$$

$$\vec{g} = \frac{\vec{F}_g}{m_0}$$

E-Field

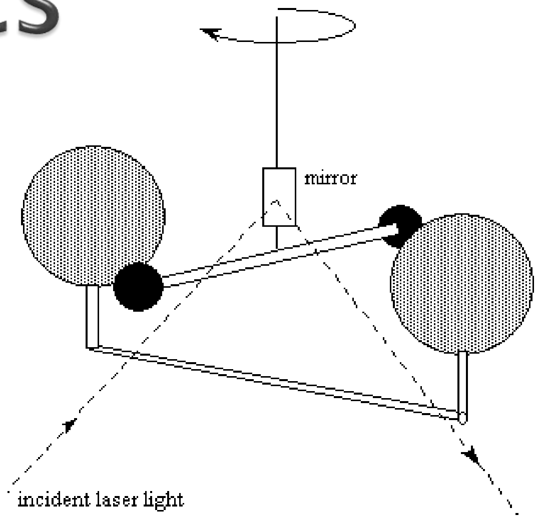
$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r}$$

E-Field Calculus

$$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{r^2}$$

History of electrostatics

- Basic principles established for macroscopic objects in late 1700s
  - Analysis of interactions between charged objects
  - Phenomenological model
- Applicable over 25 orders of magnitude in length:
  - Earth's magnetic field (10^7 m)
  - Coulomb experiments (10^0 m)
  - α particle scattering (Rutherford, 10^{-13} m)
  - Electron–positron scattering (QED, 10^{-18} m)



Schematic of Cavendish apparatus used by Coulomb. Picture from http://www.fas.harvard.edu/~scdiroff/lds/NewtonianMechanics/CavendishExperiment/CavendishExperiment.html

Coulomb's law

- Every model uses Coulomb's law (somewhere)
- Phenomenological model circa 1785 for charge-charge interactions in a vacuum
- Relates potential to charge for *homogeneous* dielectric materials
- Provides *superposition* of potentials
- Assumptions:
  - Vacuum
  - Point charges
  - No mobile ions
  - Infinite boundaries

$$u(r) = \frac{q_1 q_2}{4\pi\varepsilon_0 r}$$

Electrostatics uses a bewildering number of unit conventions

- Basic unit system can be identified by looking for the “$4\pi\epsilon_0$”

Charge (C)

$$u(r) = \frac{q_1 q_2}{4\pi\varepsilon_0 r}$$

Energy (J)

Vacuum permittivity (8.854×10^{-12} $C^2\ J^{-1}\ m^{-1}$)

Distance (m)

SI units

- Charge: C
- Energy: J
- Distance: m
- Potential: V = J C^{-1}
- Capacitance: F = C V^{-1}

$$e_c = 1.602 \times 10^{-19}\ \text{C}$$

$$\varepsilon_0 = 8.854 \times 10^{-12}\ \text{C}^2\ \text{J}^{-1}\ \text{m}$$

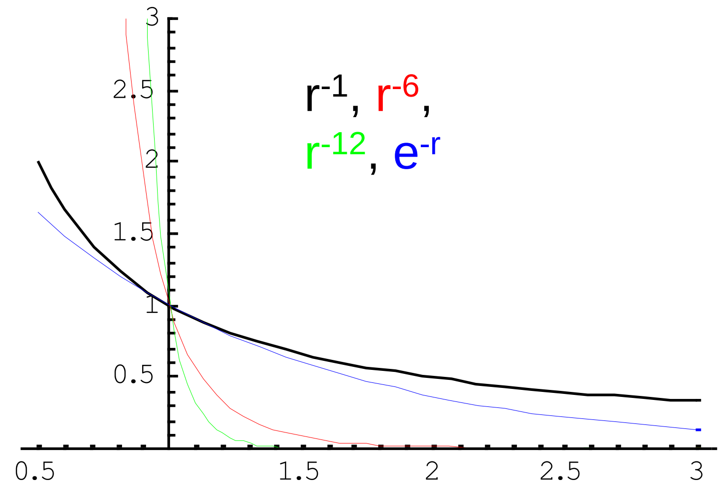
$$N_{av} = 6.022 \times 10^{23}\ \text{mol}^{-1}$$

$$\frac{N_{av} e_c^2}{4\pi\varepsilon_0} = 1.386 \times 10^{-4}\ \text{J m mol}^{-1}$$

$$u(r) = \frac{z_1 z_2}{r} \times \left(1.386 \times 10^{-4}\ \text{J m mol}^{-1}\right)$$

Charge interactions are long-ranged

- Decays much more slowly than other interactions
- The Coulomb potential cannot be integrated over an infinite domain
- Sums related to electrostatic interactions (see next example) are *conditionally convergent*



$$\lim_{R\to\infty}\left(\int_{\|\mathbf{x}\|<R}\frac{1}{\|\mathbf{x}\|}d\mathbf{x}\right)=\lim_{R\to\infty}\left(\int_0^R\frac{1}{r}4\pi r^2dr\right)$$

$$=\lim_{R\to\infty}\left(2\pi R^2\right)$$

$$=\infty$$

$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}=1-\left(\frac{1}{2}-\frac{1}{3}\right)-\left(\frac{1}{4}-\frac{1}{5}\right)-\cdots<1$$

$$=\left(1+\frac{1}{3}+\frac{1}{5}\right)-\frac{1}{2}+\left(\frac{1}{7}+\frac{1}{9}+\frac{1}{11}+\frac{1}{13}+\frac{1}{15}\right)-\frac{1}{4}+\cdots=\frac{3}{2}$$

$$=\log(x+1)\big|_{x=1}=\log(2)$$

Electrostatic interactions are strong

- Electrostatic interactions are much stronger than most other non-bonded interactions; e.g., gravitational
- Charge imbalance has serious energetic penalties
  - Lightning
  - Static electric "shock" = ~1000 charges
- Solution charge imbalance
  - ~1000 charges
  - 1 part in 10^{21}
- Electroneutrality of a *macroscopic* solution is a reasonable assumption

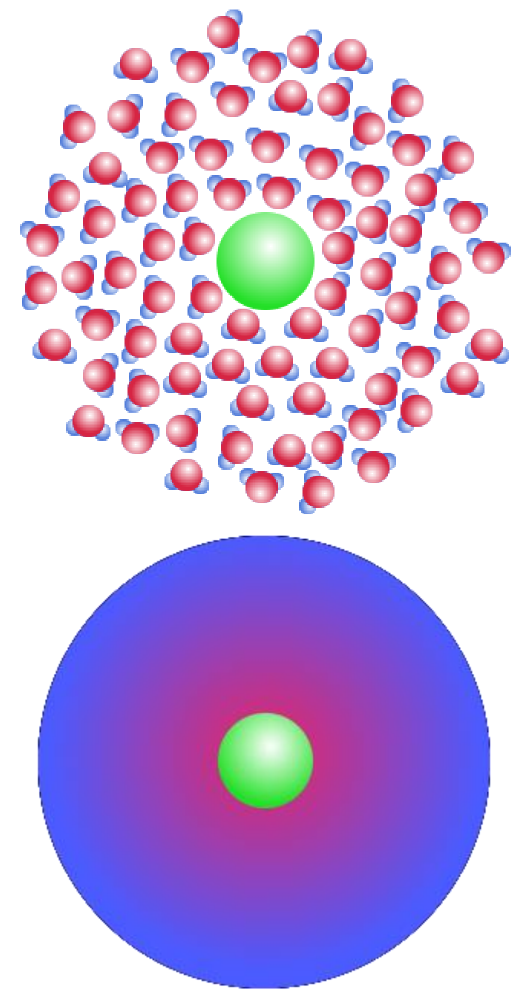
$$U_{\text{grav}} = -\frac{Gm_p^2}{r} \approx \frac{1.87\times10^{-64}}{r}\ \text{J m}$$

$$U_{\text{elec}} = \frac{e_c^2}{4\pi\varepsilon_0 r} \approx \frac{2.30\times10^{-28}}{r}\ \text{J m}$$

$$\left|\frac{U_{\text{elec}}}{U_{\text{grav}}}\right| \approx 1.23\times10^{36}$$

Electrostatics in dielectric media

- A continuum dielectric medium
  - Has no atomic detail
  - Is related to *polarization* of the medium: redistribution of charges
  - Responds *linearly* and *locally* to dampen an applied field
  - Is characterized by a dielectric tensor
  - Reduces the strength of electrostatic interactions relative to a vacuum



Electrostatics in dielectric media

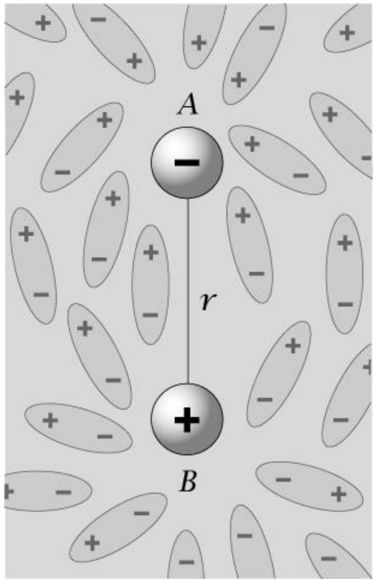
- An *isotropic* dielectric continuum exhibits the same response in all directions
  - The dielectric tensor can be reduced to a scalar
  - For a homogeneous isotropic Coulomb's law takes a very simple, scaled form

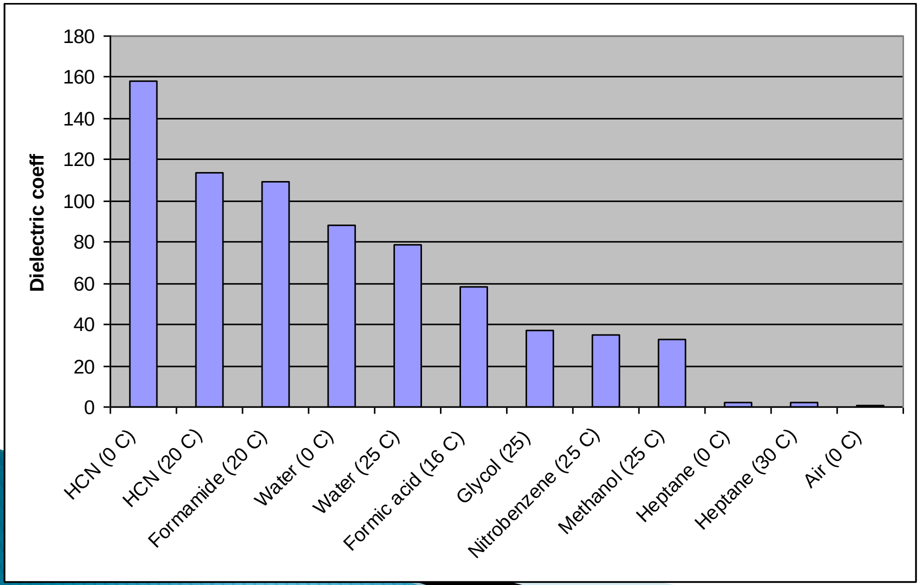
$$u(r) = \frac{q_1 q_2}{4\pi\varepsilon_0 D r}$$

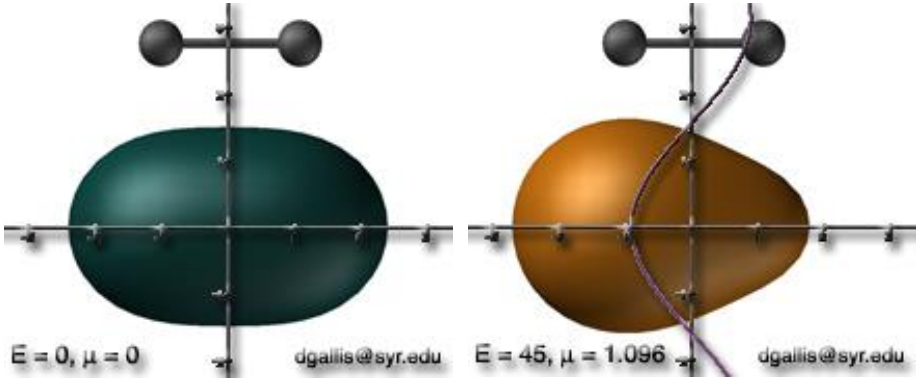
Dielectric coefficient (unitless)

Dielectric coefficients

- Several contributions to polarizability
  - Reorientation of permanent dipole moment
  - Molecular electronic and nuclear polarizability
  - Hydrogen bonding networks







Electrostatic forces

- Force is the negative gradient of the potential
- *Assume* all other terms are constant (homogeneous medium)
- Force is vector-valued

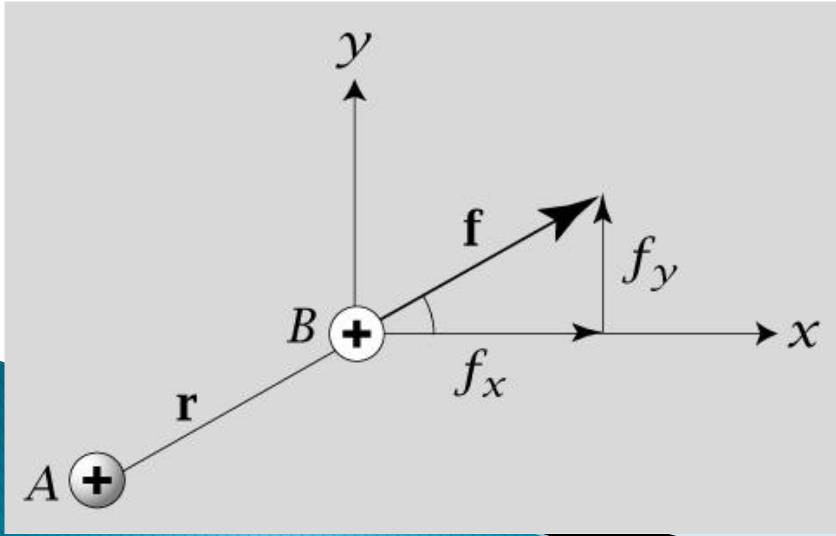
$$f_i = -(\nabla u)_i$$

$$\mathbf{f} = \frac{q_A q_B}{4\pi\varepsilon_0 D r^2}\frac{\mathbf{r}}{r}$$

$$f_r = \frac{q_A q_B}{4\pi\varepsilon_0 D r^2}$$

$$f_x = \frac{q_A q_B}{4\pi\varepsilon_0 D r^2}\cos\alpha$$

$$f_y = \frac{q_A q_B}{4\pi\varepsilon_0 D r^2}\sin\alpha$$



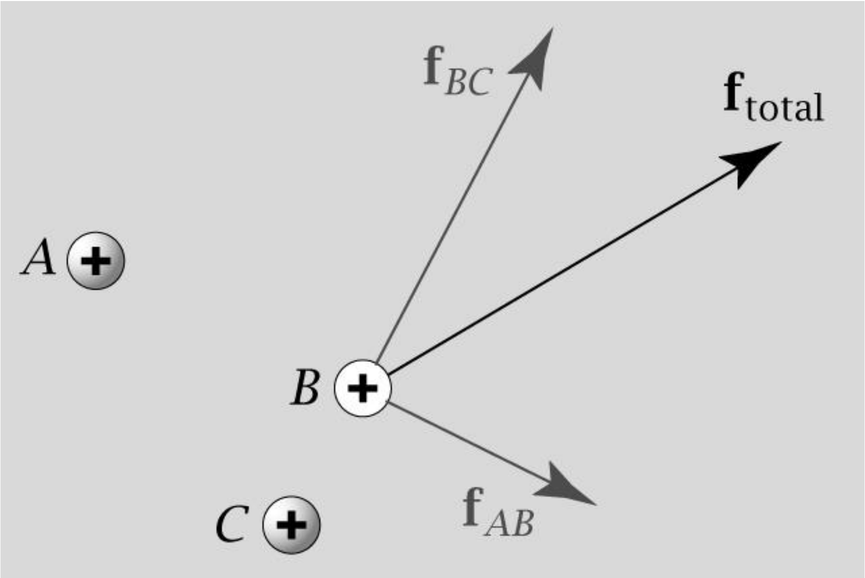
Force on charge B due to charge A.

Electrostatic superposition

- For a *homogeneous system*…
  - Total electrostatic potential is the sum of individual electrostatic potentials
  - Total electrostatic force is the sum of individual electrostatic forces
- This works for arbitrary charge distributions
- This is because Coulomb's law is a "Green function" for a particular partial differential equation (coming up…)

$$U = \frac{q_{\text{ref}}}{4\pi\varepsilon_0 D}\sum_i \frac{q_i}{\|\mathbf{x}_i - \mathbf{x}_{\text{ref}}\|}$$

$$\mathbf{F} = \frac{q_{\text{ref}}}{4\pi\varepsilon_0 D}\sum_i \frac{q_i}{\|\mathbf{x}_i - \mathbf{x}_{\text{ref}}\|^2}\frac{\mathbf{x}_i - \mathbf{x}_{\text{ref}}}{\|\mathbf{x}_i - \mathbf{x}_{\text{ref}}\|}$$

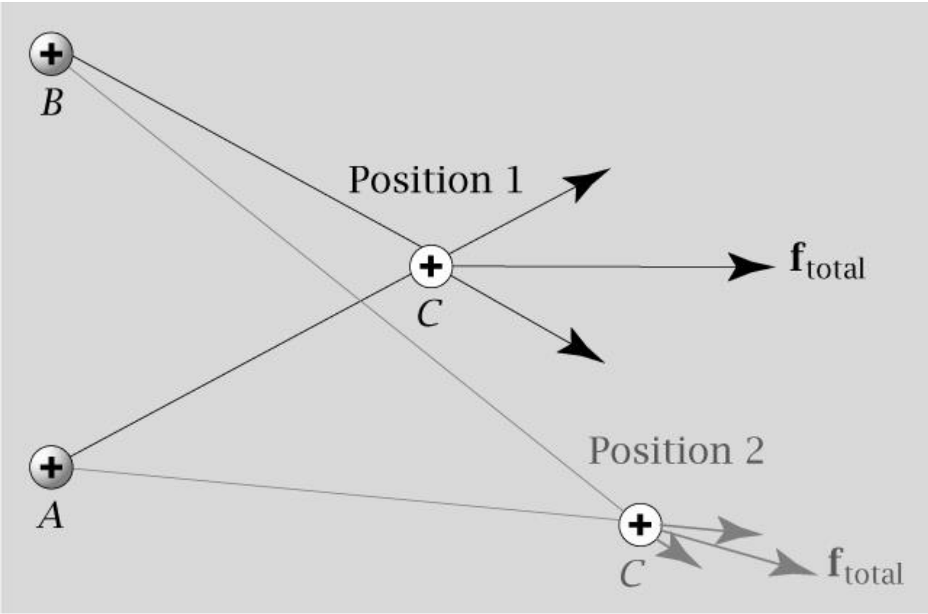


Electrostatic fields and potentials

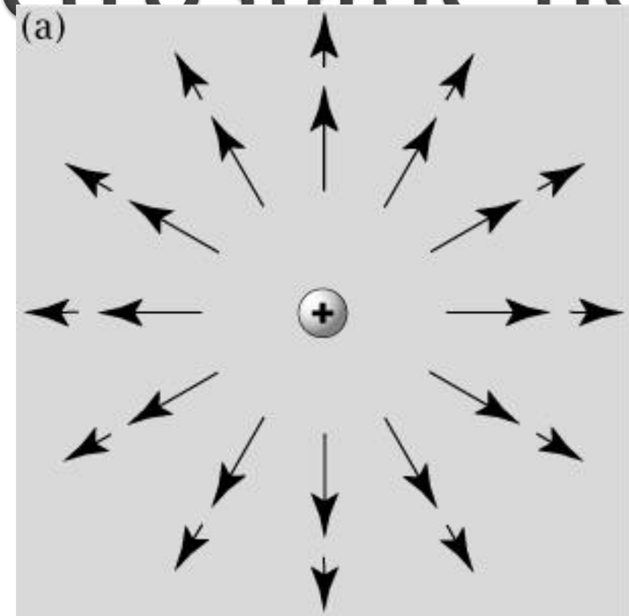
- Potential:
  - What is the energy of placing a unit charge at position **x**?
  - A scalar-valued function
  - Factoring charge (C) out of energy (J) gives units of V = J C^{-1}
- Field:
  - What is the force experienced by a unit charge at position **x**?
  - A vector-valued function
  - Factoring charge (C) out of force (N = J m^{-1}) gives units of N C^{-1}
- Superposition applies: potentials and forces can be added
- Purpose: a good way to represent the electrostatics of a charge distribution

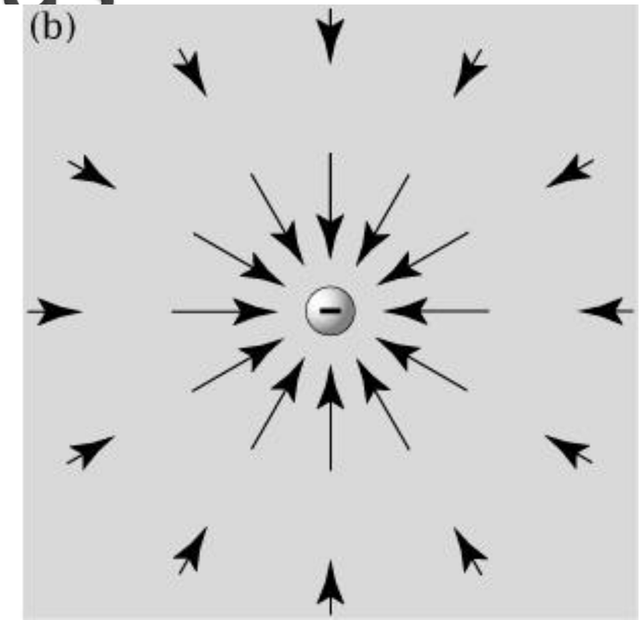
$$\psi(\mathbf{x}) = \frac{1}{4\pi\varepsilon_0 D}\sum_i \frac{q_i}{\|\mathbf{x}_i - \mathbf{x}\|}$$

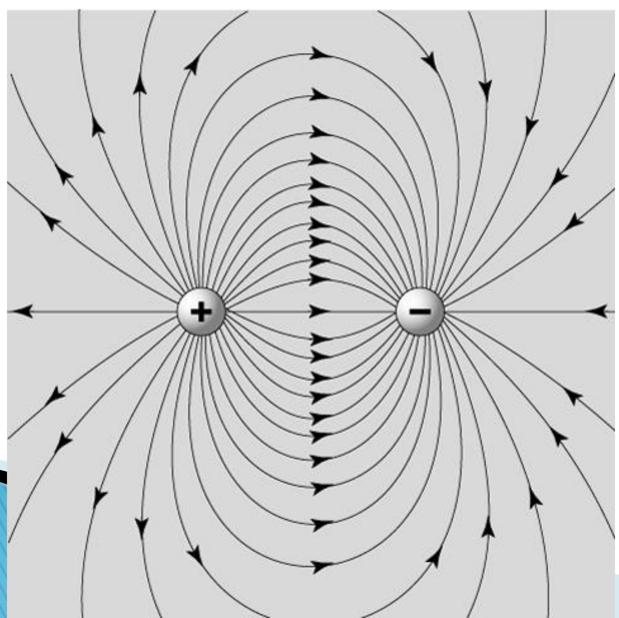
$$\mathbf{E}(\mathbf{x}) = \frac{q}{4\pi\varepsilon_0 D}\sum_i \frac{q_i}{\|\mathbf{x}_i - \mathbf{x}\|^2}\frac{\mathbf{x}_i - \mathbf{x}}{\|\mathbf{x}_i - \mathbf{x}\|}$$

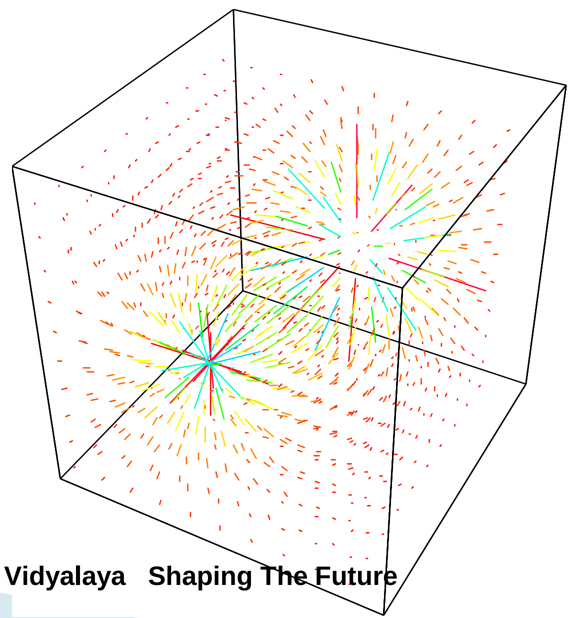


Electrostatic fields









Electric field flux

- Flux: the amount of "stuff" passing through surface
  - Concentration
  - Fluid flow
  - Electric field
- Fluxes arise from
  - Sources: positive charges
  - Sinks: negative charges
- Electric field flux: integral of electric displacement over a surface

$$\Phi = \int_{\partial\Omega} \underbrace{D\mathbf{E}}(\mathbf{s}) \cdot d\mathbf{s}$$

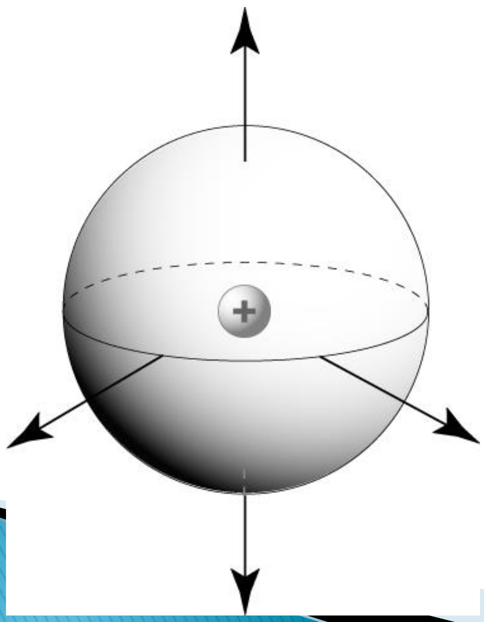
Boundary surface of volume Ω

Electric displacement

Jacobian; points in surface normal direction

Field flux: point charge in a sphere

- Point charge has spherically-symmetric field
- Field is constant on sphere surface
- Flux is independent of sphere diameter



$$\Phi = \int_0^{\pi}\int_0^{2\pi} DE(r)\, r^2 \sin\theta\, d\theta\, d\psi$$

$$= DE(r)\, 4\pi r^2$$

$$= D\frac{q}{4\pi\varepsilon_0 D r^2} 4\pi r^2$$

$$= \frac{q}{\varepsilon_0}$$

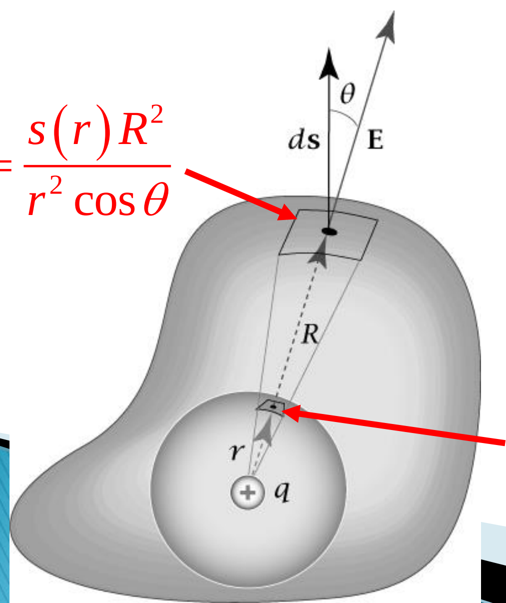
Field flux: point charge in a balloon

- Consider another "outer" surface that surrounds an "inner" sphere
- The outer surface can have any shape
- The fluxes through any (arbitrarily small) portion of the outer and inner surfaces can be calculated
- These surface portions can be related
- The fluxes through the two surfaces are the same!

$$\Phi_{\text{in}}[s] = DE(r)s(r) = \frac{qs(r)}{4\pi\varepsilon_0 r^2}$$

$$\Phi_{\text{out}}[S] = DE(R)S(R)\cos\theta = \frac{qS(R)\cos\theta}{4\pi\varepsilon_0 R^2}$$

$$S(R) = \frac{s(r)R^2}{r^2\cos\theta}$$



$$\frac{\Phi_{\text{out}}[S]}{\Phi_{\text{in}}[s]} = \frac{\dfrac{qS(R)\cos\theta}{4\pi\varepsilon_0 R^2}}{\dfrac{qs(r)}{4\pi\varepsilon_0 r^2}} = \frac{S(R)}{s(r)}\frac{r^2}{R^2}\cos\theta = \frac{\dfrac{s(r)R^2}{r^2\cos\theta}}{s(r)}\frac{r^2}{R^2}\cos\theta = 1$$

Gauss' law

- The integral of field flux through a closed, simple surface is equal to the total charge inside the surface
- This is true for *both* homogeneous and inhomogeneous dielectric media
- This generalizes to other charge distributions

$$\Phi = \oiint_{\partial\Omega} D\mathbf{E}(\mathbf{s}) \cdot d\mathbf{s}$$

$$= \oiint_{\partial\Omega} D\left(\sum_i \mathbf{E}_i(\mathbf{s})\right) \cdot d\mathbf{s}$$

$$= \sum_i \oiint_{\partial\Omega} D\mathbf{E}_i(\mathbf{s}) \cdot d\mathbf{s}$$

$$= \frac{1}{\varepsilon_0} \sum_i q_i$$

$$\Phi = \oiint_{\partial\Omega} D\mathbf{E}(\mathbf{s}) \cdot d\mathbf{s}$$

$$= \frac{1}{\varepsilon_0} \int_{\Omega} \rho(\mathbf{x})\, d\mathbf{x}$$

Field from a line charge

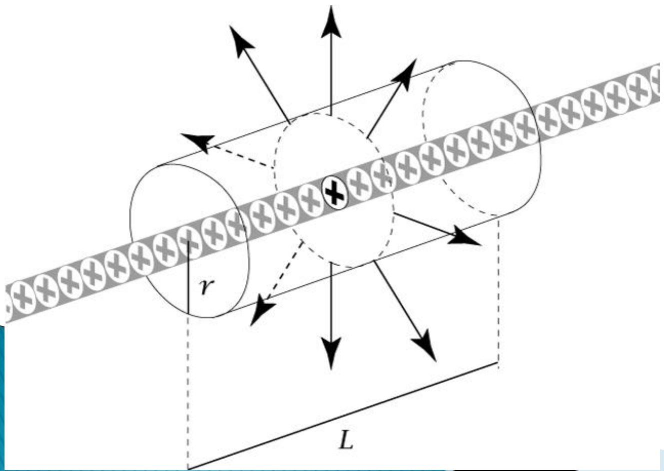
- Suppose we have
  - Homogeneous medium
  - Line of length *L*, where *L* is "very big" (radial symmetry)
  - Linear charge density of λ
- What is the field at distance *r* from the source?
- Compute the flux through a cylindrical surface
- Calculate the enclosed charge
- Use Gauss' Law

$$\Phi = \oint_{\partial(\text{cylinder})} D\mathbf{E}(\mathbf{s}) \cdot d\mathbf{s}$$

$$= \int_0^{2\pi} \int_0^L DE(r) r\,dz\,d\theta$$

$$= 2\pi r L D E(r)$$

$$\Phi = \int_{\text{cylinder}} \rho(\mathbf{x})\,d\mathbf{x} = \frac{q}{\varepsilon_0} = \frac{\lambda L}{\varepsilon_0}$$



$$2\pi r L D E(r) = \frac{\lambda L}{\varepsilon_0}$$

$$E(r) = \frac{\lambda}{2\pi \varepsilon_0 D r}$$

Field from a charged plane

- Suppose we have
  - Homogeneous medium
  - Surface of area *A*, where *A* is "very big" (one dimensional)
  - Surface charge density of σ
- What is the field at distance *r* from the source?
- Compute the flux through a "pillbox"
- Calculate the enclosed charge
- Use Gauss' Law

$$\Phi = \oint_{\partial(\text{pillbox})} D\mathbf{E}(\mathbf{s}) \cdot d\mathbf{s}$$

$$= 2\int_0^{2\pi}\int_0^R DE(z)\, r\,dr\,d\theta$$

$$+ \int_0^{2\pi}\int_0^z D \cdot 0 \cdot r\,dz\,d\theta$$

$$= 4\pi R^2 DE(z) = 2ADE(z)$$

$$\Phi = \int_{\text{pillbox}} \rho(\mathbf{x})\, d\mathbf{x} = \frac{q}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0}$$

$$2ADE(z) = \frac{\sigma A}{\varepsilon_0}$$

$$E(z) = \frac{\sigma}{2\varepsilon_0 D}$$