

Gauss' Law

A new look at Coulomb's law

When there is much symmetry it is best to use Gauss' law, which is equivalent to Coulomb's law in electrostatics.

ATUL VIDYALAYA

A Gaussian surface is a hypothetical closed surface.

Gauss' law (GL) relates the electric fields at points on a Gaussian surface to the net charge enclosed by that surface.



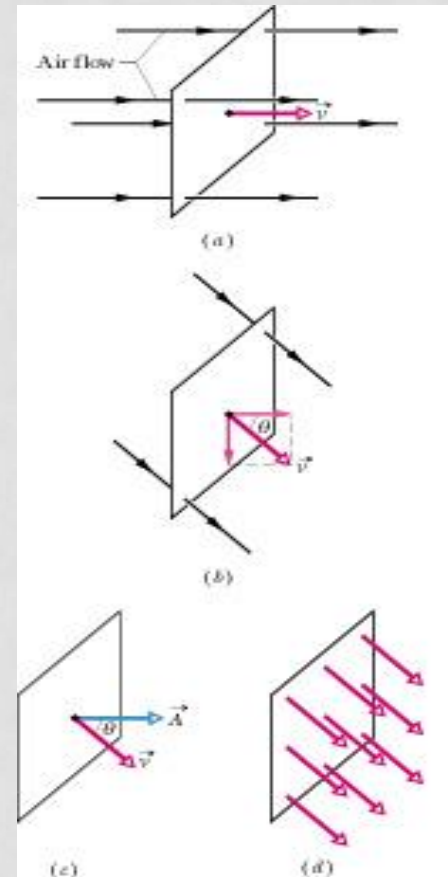
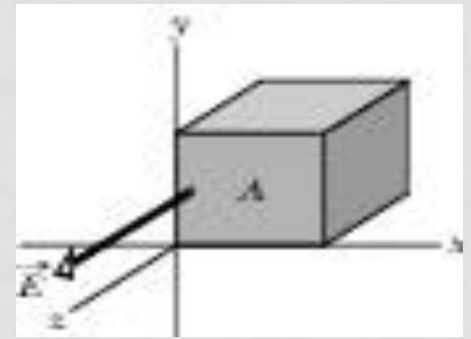
Flux (Φ):

Flux through an area is the product of an area and the (vector) field across that area.

We define the vector area (\vec{A}) as being a vector whose magnitude is equal to the area and its direction is normal to the plane of the area.

The flux (Φ) of a vector field \vec{v} through this area \vec{A} is:

$$\Phi = v A \cos \theta = \vec{v} \cdot \vec{A}$$



Flux for an electric field through a closed surface:

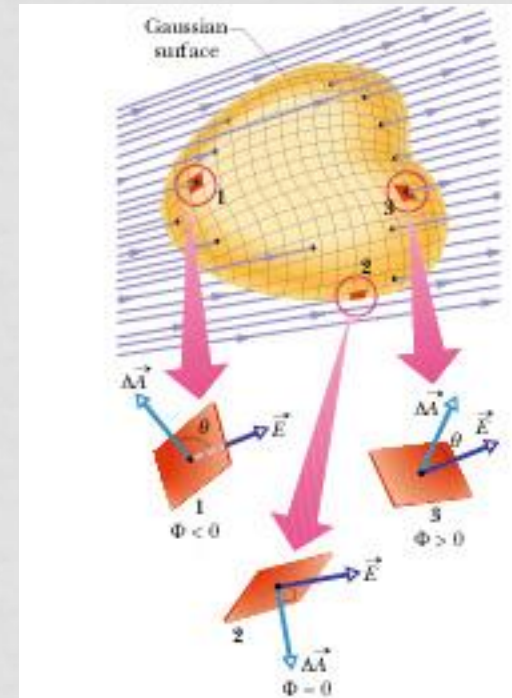
$$\Phi \cong \sum \vec{\mathbf{E}} \cdot \vec{\Delta\mathbf{A}}$$

Φ can be positive (when there is a net flux outward), zero (when there is no net flux) or negative (when there is a net flux inward).

$$\Phi = \oint \vec{\mathbf{E}} \cdot \vec{d\mathbf{A}}$$

where the integral is over the closed surface.

Obviously, the electric flux Φ through a Gaussian surface is proportional to the net number of electric field line passing through the surface.



Gauss' law:

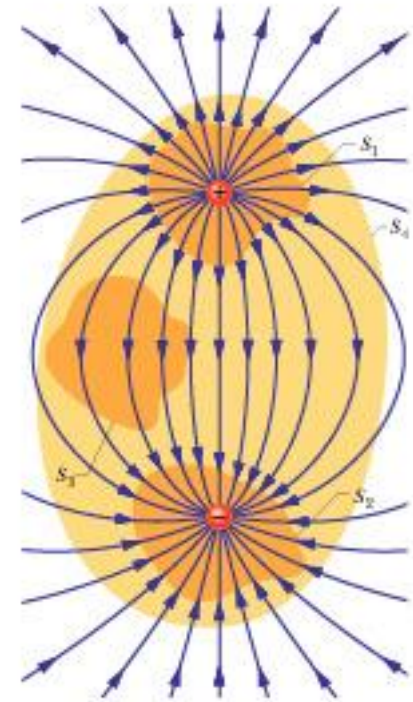
$$\epsilon_0 \Phi = q_{\text{enc}}$$

Apply Gauss's law to the four Gaussian surfaces of the figure.

We will see that Gauss' law makes it easy for us to calculate the electric field in situations containing (enough) symmetry.

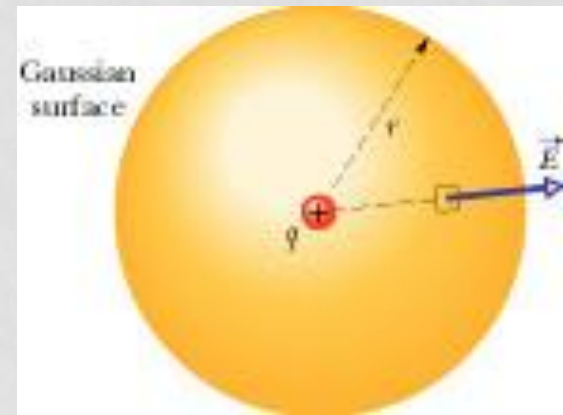
Solve sample problem 24-3:

A cube centered at the origin with sides perpendicular to the coordinate axes has 1.40 m edges and is located in a uniform electric field: $-3.00 \mathbf{i} + 4.00 \mathbf{k}$. Find the flux through the right face (with $y = 1.40$ m). What if the field was $-2.00 \mathbf{j}$?



Gauss' law and Coulomb's law:

Each can be derived from the other. Let's derive CL from GL. Take spherically symmetric Gaussian sphere about a point charge and apply GL.



Gauss' law and the charged isolated conductor:

(I) An isolated solid conductor:

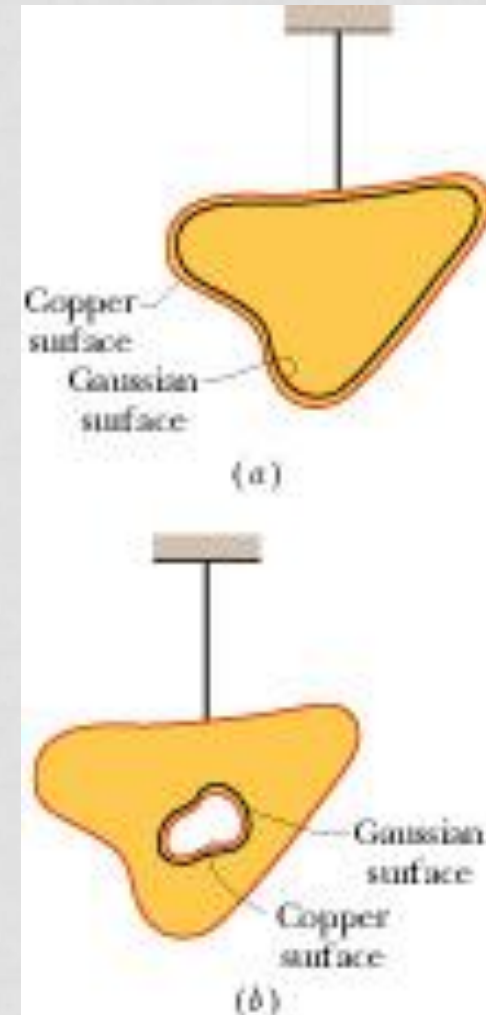
$$E_{\text{inside}} = 0 \quad (\text{why?})$$

$$Q_{\text{inside}} = 0 \quad (\text{by applying GL})$$

So all charges on a conductor can reside *only on the surface* of the conductor.

(II) An isolated conductor with a cavity:

There can be no net charge on the cavity walls. (why?)



(III) The conductor removed:

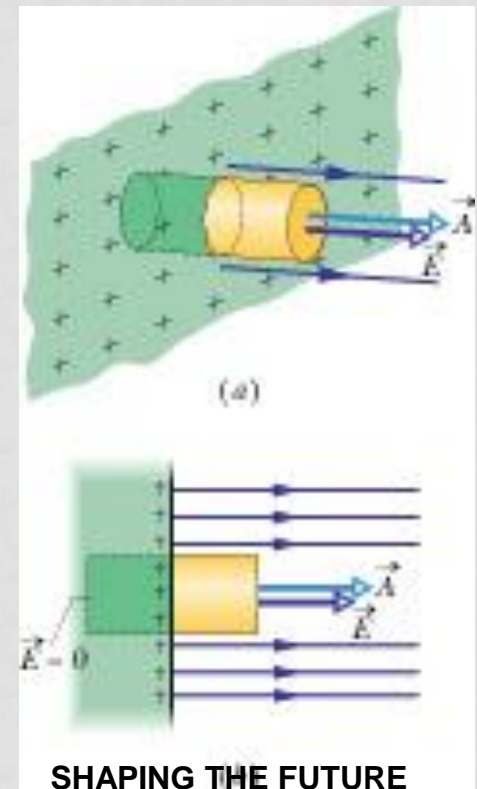
This is equivalent to enlarging the cavity until there are only the charges. No field inside the thin shell of charge; field unchanged for external points.

(IV) The field just outside the conductor:

The electric field is normal to the surface and has a magnitude:

$$E = |\sigma| / \epsilon_0$$

The direction of E is away from the surface if $\sigma > 0$ and toward the surface if $\sigma < 0$.



Applying Gauss' law: Cylindrical symmetry:

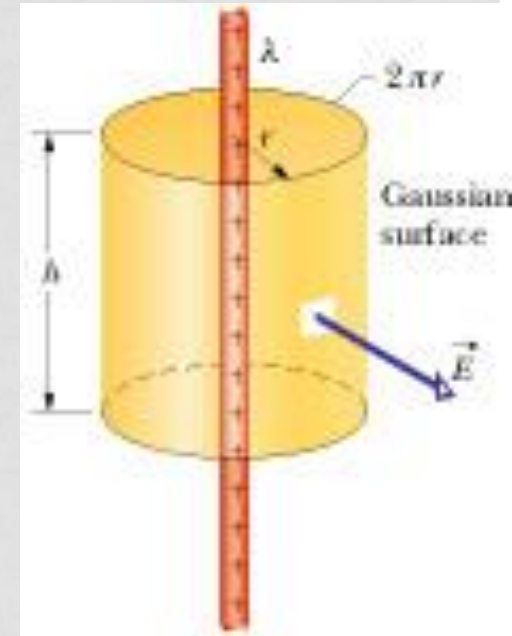
What is the magnitude of the field at points outside the cylinder?

What is the direction?

Interaction: What is the magnitude of the field at points inside the cylindrical string assuming:

- (a) the string is conducting, and
- (b) the string is insulating and uniformly charged?

How would things differ if it were not an infinite cylinder?



The parallel plate capacitor [two conducting plates of area A and charges Q and -Q]:

The field between the plates has a magnitude

$$E = Q/(\epsilon_0 A)$$

and points from the positively charged plate to the negatively charged plate.

